



Tutorial: Map projections and coordinate reference systems



Map projections and coordinate reference systems

In this course unit the problem of mapping the curved earth figure to a plane or another substitute figure are explained (map projections). Common coordinate reference systems are described.

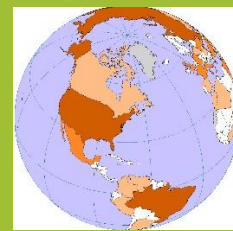
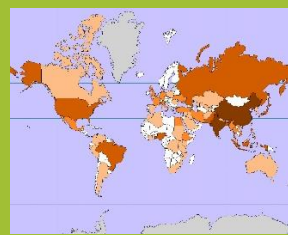


Image sources: Bill (2016, p. 184)

Maps net designs

The earth has a very complex figure which needs to be approximated for mapping purposes. The simplest substitution is the *sphere*. A mathematically more complex description is the *ellipsoid*. The actual shape resembles a potato, however, which is named a *geoid*. Since an irregular body is not very suitable for calculations, in most cases an ellipsoid is used as a geometric approximation. There are different ellipsoids available, which are used depending on the mapped area. Since the ellipsoid always only approximates the earth, smaller deviations occur in the figure. To minimize these, there are locally adapted ellipsoids, such as the *Bessel* ellipsoid (for Germany) or the *Krassowski* ellipsoid (for Russia). The dimensions and positions of these ellipsoids are such that they reproduce the depicted section of the earth's surface particularly well. However, they are not suitable for global observations. Geocentrically supported ellipsoids, such as the *WGS 84* ellipsoid, are used for this purpose. Geocentric means that the center of the ellipsoid and the center of the earth are identical.

As curved bodies, the ellipsoid cannot easily be mapped on a plane, just as one cannot wrap a potato in a sheet of paper without wrinkles forming. A simple example should illustrate the idea of such map projections.

Illustration example: Let us assume that the earth is a glass sphere with a light bulb at its centre. On the sphere, the continents and the network of geographic coordinates (meridians and parallels) are marked as black lines. If you now turn on the light bulb and hold a sheet of paper to the sphere, the lines cast shadows. If you draw these shadows, you get a map. Usually the projection to the map plane is somewhat more complex, but the basic idea is the same.

The position of the light bulb can vary as can the position and shape of the sheet of paper on which it is projected. Depending on the position of the projection center (= "bulb"), a distinction is made between orthographic, stereographic and gnomonic projection (see Figure 1).

The *orthographic projection* is a parallel projection. The center of the projection is infinite. The projection centre of a *stereographic projection* is exactly opposite the point of contact of the image plane. A connecting vector between the point of contact and the projection centre leads

through the centre of the sphere. In *gnomonic projection*, on the other hand, the projection center is located in the center of the sphere.

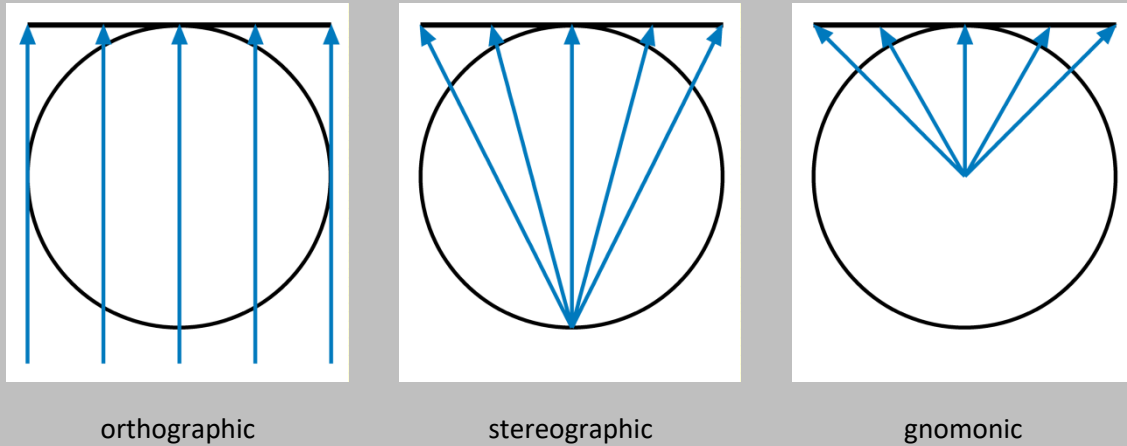


Figure 1: Position of the projection center

Map projections lead to certain properties of the map. A map can never be distortion-free. However, certain characteristics such as *fidelity in length*, *fidelity in angle* and *fidelity in area* can be partially achieved. Angular fidelity means that shapes are preserved, as are course angles. This property is important for mapping as well as navigation at sea or in the air. Faithful images do not contain surface distortion. Fidelity of angle and fidelity of area are mutually exclusive. Tissot's indicatrix is used to illustrate the properties of an image. Tissot's indicatrix shows a circle that is enlarged or distorted to an ellipse depending on the properties of the figure (see Figures 2 and 3).

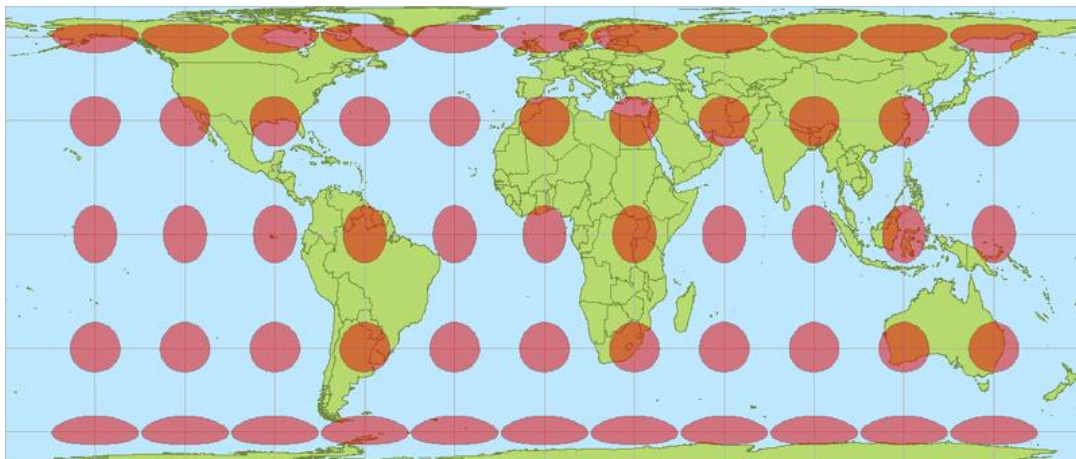


Figure 2: Area fidelity represented by Tissot's indicatrix

(Source: Stefan Kühn, www.wikipedia.de, 05.08.13)

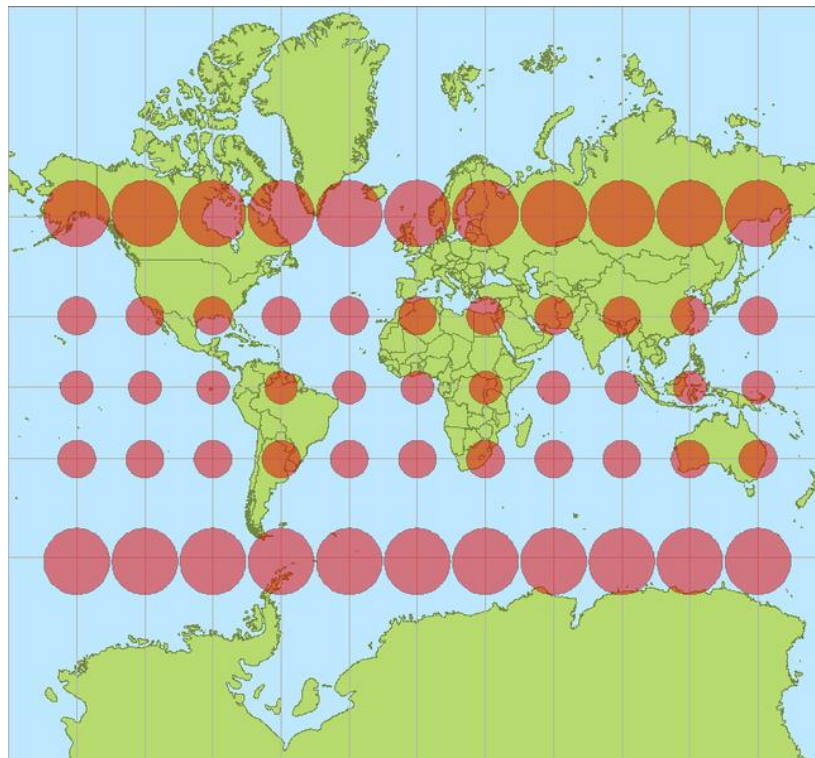


Figure 3: Angular fidelity represented by Tissot's indicator matrix

(Source: Stefan Kühn, www.wikipedia.de, 05.08.13)

The stereographic projection results in an azimuthal image (projection on one plane) with a true angle image. The gnomonic projection, on the other hand, is neither angle- nor area-faithful for imaging on a plane. Mediating designs try to minimize the overall distortion.

In addition to the plane, cones and cylinders (see Figure 4) are also used as imaging bodies (= projection surface). Once the image of the earth has been projected onto the imaging body, the cone and cylinder can simply be unwound into the plane. This is how the map is created. The cone is cut open from the lower edge in a straight line to the tip of the cone and the cone shell is then spread out. The cylinder jacket is cut from the lower edge to the upper edge in a straight line and also rolled out.

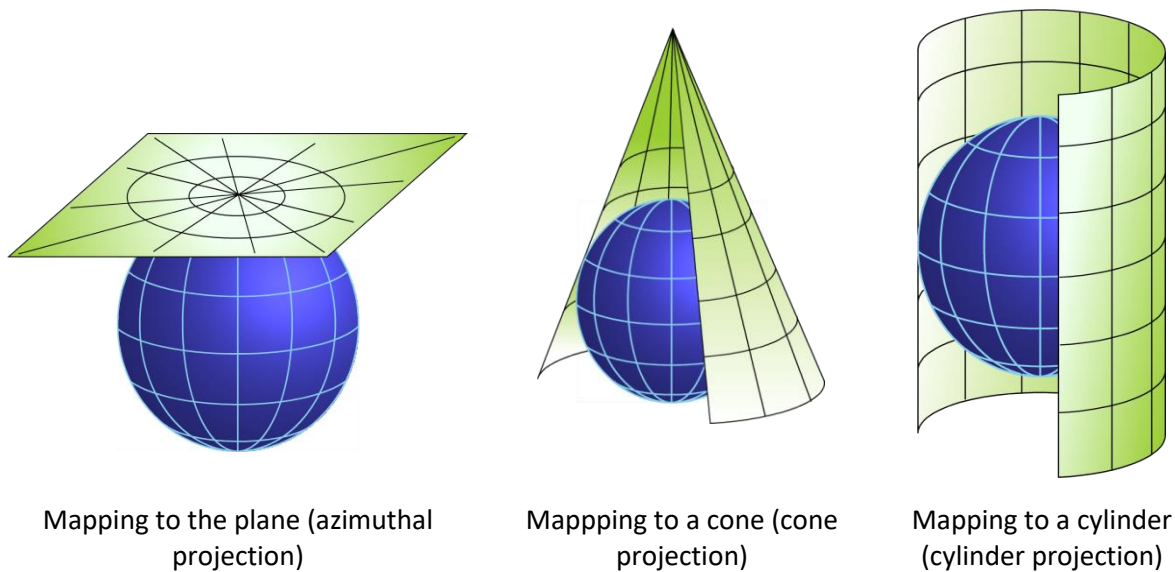


Figure 4: Figure body

The bodies can take different positions with respect to the sphere or the ellipsoid (see Figure 5).

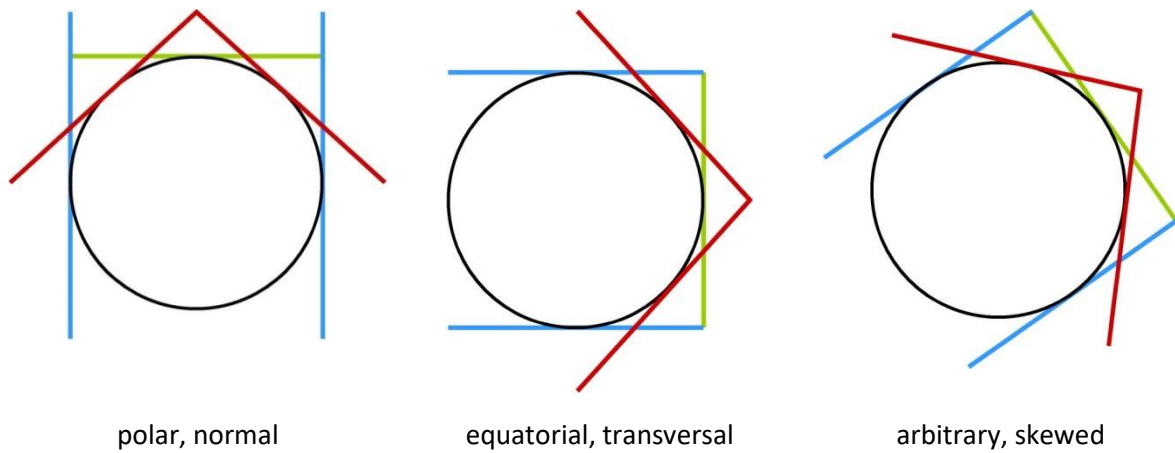


Figure 5: Position of the imaging bodies

The imaging bodies can touch or intersect the earth. However, they may also be non-contact (see Figure 6). The contact or intersection lines (e.g. the intersection meridians in the UTM figure) are mapped undistorted.

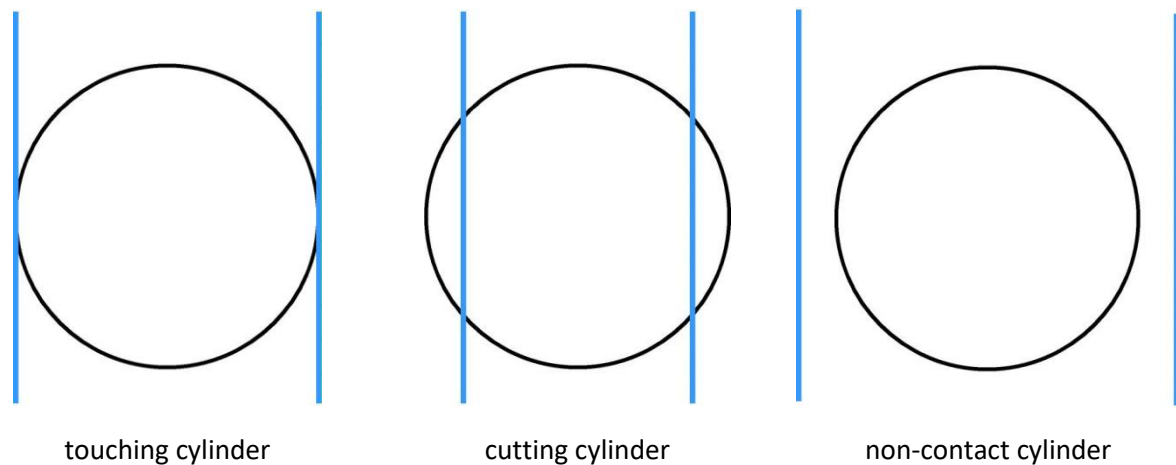


Figure 6: Position of the imaging bodies

The choice of the map net design depends primarily on the area to be mapped. In general, small-scale maps that represent a large area and use a polar map body should be used:

- for polar regions: Azimuth projection
- for a mid-latitude area: conical projection
- for an area along the equator: cylindrical projection in normal position

The map net designs are divided into real and fictitious designs. This classification is based on the illustration of meridians and parallels of latitude. Genuine designs depict meridians as tufts of straight lines or as a group of parallel lines. The circles of latitude are depicted as concentric circles or as a second group of parallel straight lines. The orthogonality between meridians and parallels of latitude is also preserved in the illustration. In fictitious designs, on the other hand, meridians are depicted as any curves. The orthogonality between the coordinate lines of the geographic coordinate network is lost in the illustration.

Coordinate reference systems

There are a number of different coordinate systems that can be used either locally or globally. Almost every country has developed its own national coordinate system and published its official topographic maps. In the course of the unification of topographic maps on a global level, the change from the previous *Gauß-Krüger system* to the new *UTM system* took place in the official maps of Germany. In the transition phase, both systems are indicated on the maps. In the future, the Gauß-Krüger coordinates are to be omitted. In addition to these two Cartesian (flat, rectangular) coordinate systems, geographical coordinates are also always indicated.

Geographical coordinates

Geographical coordinates refer to the figure of the earth. The coordinate lines are called meridians and latitudes. Meridians run through the north and south poles and intersect the latitudes at right angles. The latitudes are not great circles except for the equator, i.e. their center is not identical with the center of the earth. Geographical coordinates are described in longitude (meridians) and latitude (latitudes) values. The unit of measurement is degrees ($^{\circ}$), where 60 geographical seconds ($''$) correspond to 1 geographical minute ($'$) and $60' = 1^{\circ}$. For a globally unambiguous positioning, the meridians are provided with the data w.L. (western longitude) or e.L. (eastern longitude) and the latitude circles with n.L. (northern latitude) and s.L. (southern latitude). A zero meridian serves as a dividing line between eastern and western longitude. There used to be several of these prime

meridians until the Greenwich prime meridian was agreed in 1884. The equator marks the separation between the northern and southern hemispheres.

Geographical coordinates are given at the corners of the sheets of official topographic maps. To read the intermediate values, the sections on the inner map frame marked alternately bright and dark are used. The geographical coordinates determine the map sheet cut (the delimitation of the individual map sheets) for the so-called degree division maps (TK10 to TUK200 in Germany). They serve only for orientation. They are less suitable for precise navigation in large scales (e.g. when hiking).

Gauss-Krüger coordinates

The Gauss-Krüger coordinate system (GK coordinates) is a Cartesian coordinate system. The GK-coordinates are called right (Rechtswert) or high value (Hochwert). The principle of the GK-coordinates consists in mapping a strip on both sides of a meridian in right-angled coordinates. The Gauß-Krüger image uses a transverse-axis cylinder that touches the earth's body along a meridian (see Figure 7). This meridian, the middle meridian of a so-called meridian strip, is depicted true to length. The further one moves away from the meridian, the more the length fidelity decreases. The coordinates are always read parallel to the coordinate lines.

In order to keep the distortions as small as possible, the contact meridian of the cylinder (middle meridian of the meridian strip) moves according to the area of representation. The meridians used for this are fixed. Usually the distance between the meridians is 3° , more rarely 6° . The official topographic maps (TK) in Germany formerly used the Gauß-Krüger system with 3° wide meridian stripes in the DHDN system (Deutsches Hauptdreiecksnetz), which can also be found under the designations Rauenberg date and Potsdam date. On average, the distance between two middle meridians in the Gauß-Krüger- 3° meridian strip system (GK3) in Germany is 200 km.

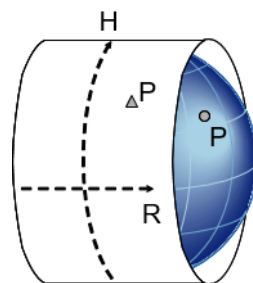


Figure 7: Mapping principle of the Gauß-Krüger coordinates (Resnik/Bill, 2018)

In the Gauß-Krüger image, the distortion increases with increasing distance from the middle meridian, i.e. from the touching cylinder. All areas of the image area are stretched to a greater or lesser extent (see Figure 8). At a distance of 100 km from the middle meridian, the deviation is 3.1

cm over a distance of 1 km. At a distance of 200 km it is already 12.3 cm on 1 km. The maximum distortion occurs at the edge of the strip. There it is about 13 cm per km (cf. STREMEL 1996).

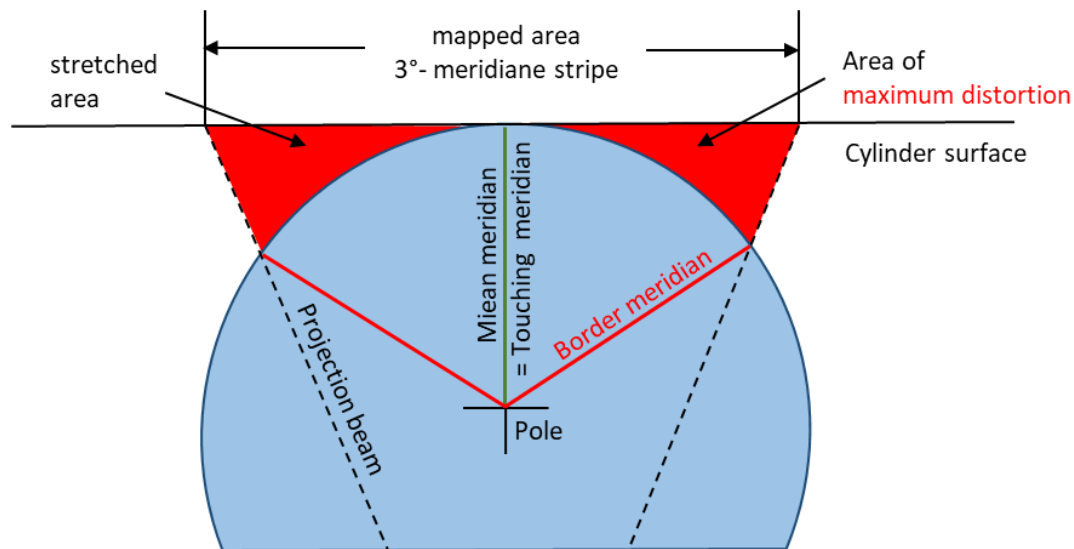


Figure 8: Distortion ranges of the Gauß-Krüger projection

The right values of the GK coordinates consist of the strip number and the distance to the corresponding middle meridian. Since this is indicated in the topographical map in km, three further digits are added to the strip code number. The strip numbers of the 3° meridiane strip system (GK3) are shown in Table 1:

Table 1: Stripe indices of the Gauß-Krüger coordinates

middle meridian (eastern longitude of Greenwich)	3°	6°	9°	12°	15°
strip code number	1	2	3	4	5

A simple rule to remember the strip number in the GK3 system is:

$$\text{Stripe index} = (\text{degrees of the middle meridian})/3$$

The reverse is true:

$$\text{Degree of the median meridian} = 3 \times \text{strip identification number}$$

To avoid negative values for points west of the mid meridian, the mid meridian is added the value 500 km. If the three-digit distance in the right value of the Gauss-Krüger coordinates is less than 500, the amount of 500 is subtracted to determine the distance of the point from the middle meridian. However, if the three-digit code is greater than 500, 500 km is subtracted from the distance. This means:

$$\text{Right value} = \text{strip number } A + \text{distance specification } B \text{ (3 digits)}$$

A: see table 1

B: if $B < 500 \Rightarrow 500 - B = \text{distance in km} \Rightarrow \text{point west of the middle meridian}$

if $B > 500 \Rightarrow B - 500 = \text{distance in km} \Rightarrow \text{point east of the middle meridian}$

Conversely, the determination of the GK coordinates of a point whose distance y (in km) and position to the meridian is known applies:

B: if point lies west of mid meridian $\Rightarrow B = 500 - y$

if point lies east of the meridian $\Rightarrow B = 500 + y$

The high value indicates the distance to the equator in km (at four-digit coordinate). Since GK coordinates are generally only used in the northern hemisphere or in Europe, there is no marking of the northern and southern hemispheres.

The reading of the GK coordinates in the sub-km range in the topographic map is done either by estimating (the coordinates) or by measuring the remaining distance to the nearest coordinate.

Note: Please note that the measured map distance must be converted to natural scale using the scale.

The remaining distance thus determined is then added to or subtracted from the distance extracted from the nearest coordinate used as a reference point (depending on the position of the point on the map to the coordinate marker).

For illustration purposes, various examples for the determination of GK coordinates and geographical coordinates on the basis of numerical values and in the official topographic map are presented.

Coordinate determination based on numerical values

Example 1: Point P has the Gauss-Krüger coordinates $R = {}^{45}35450\text{m}$, $H = {}^{54}48600\text{m}$. We are looking for the distance to the middle meridian and to the equator.

First the corresponding middle meridian is determined: $3 * 4 = 12^\circ$

The distance specification is greater than 500 000m. P therefore lies east of the middle meridian. The distance to the middle meridian is calculated:

$$535\,450\text{ m} - 500\,000\text{ m} = 35\,450\text{ m}$$

P is located 35.45 km east of the 12° meridian.

The high value indicates the distance to the equator: 5 448 600 m.

P is 5 448.6 km from the equator.

Example 2: From a point Q it is known that it lies 59 km west of the 9° meridian. Its distance from the equator is 5702 km. We are looking for the GK coordinates.

The required strip code number is $A = 9^\circ / 3 = 3$.

With the given distance $y = 59$ km and the indication that Q lies west of the meridian, the three-digit distance B can be determined.

$$B = 500\text{ km} - 59\text{ km} = 441$$

The GK coordinates we are looking for are: $R {}^{34}41$, $H {}^{57}02$.

Coordinate determination in a map (fictitious sheet of the TK25, see Figure 9)

Example 1: Often, two-digit and a few four-digit coordinate data are predominantly to be found at the edge of the TK 10 and TK 25 maps. The four-digit data are complete km data of the GK coordinates, whereby the first two digits are superscript. Within a map sheet, these first two digits (strip number + 100 km) rarely change. It is therefore not necessary to specify all positions at each coordinate marking. If one of the front positions changes, this becomes visible on the label by means of 4 positions. This form of labelling is also used for UTM coordinates, whereby the letter combination of the message grid is sometimes used instead of the zone numbers of the easting value.

Example 2: The geographical coordinates are indicated at the corners of the topographic map. In the northeast corner the values $11^{\circ}30'$ eLength and $50^{\circ}42'$ nWidth can be found. In the southwest corner are the values $11^{\circ}20'$ eL and $50^{\circ}36'$ nW. The map sheet is located in the 4th meridian strip (recognizable by the strip number 4 in the right values). For simplification it was assumed that the coordinate lines of the GK coordinates run parallel to the map border. This is not always the case!

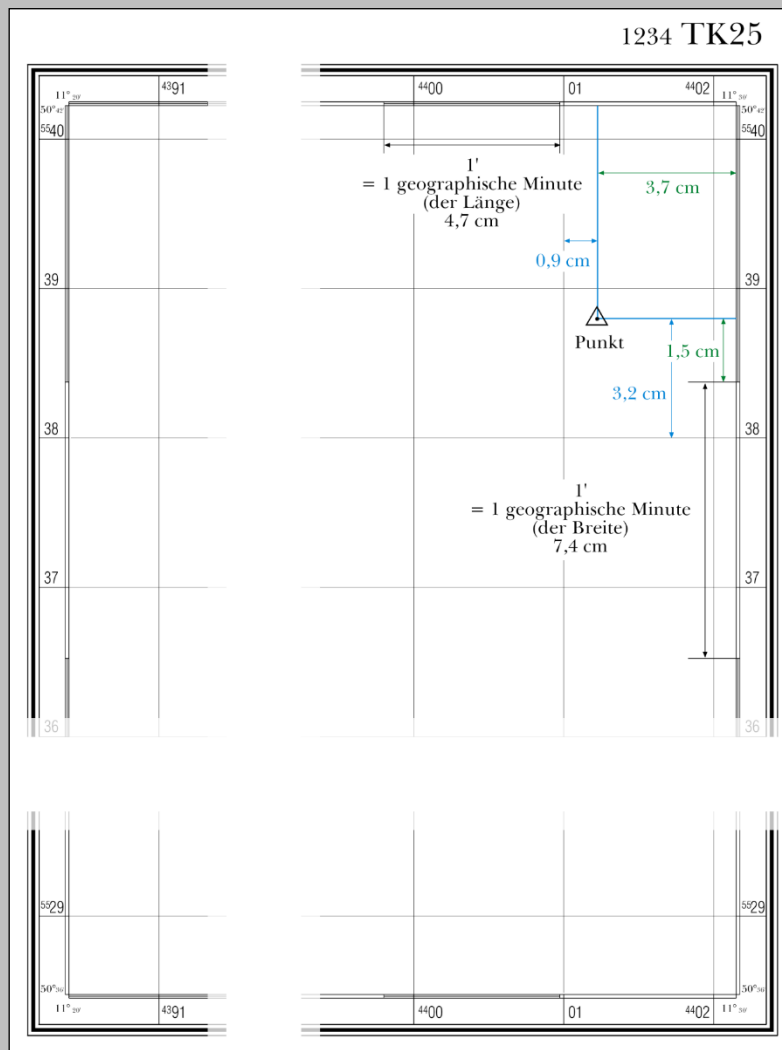


Figure 9: Coordinate determination in a topographic map

The geographical coordinates 11°30' eL (east of the point) and 50°41' nW (south of the point) were used as reference coordinates for determining the coordinates of the point. For the GK coordinates the coordinate lines 01 (actually 4401) and 38 (actually 5538) were used. The measured distances are shown in the figure. For the coordinate determination the measured distances are now set in relation to the known distances.

Geographical coordinates:

A geographical minute of longitude in the map is 4.7cm long. The measured distance is 3.7cm. We are looking for the remaining distance x:

$$x = \frac{1'}{4,7cm} \cdot 3,7cm = 0,7872' = 47''$$

The measured distance corresponds to 47'' (geographical seconds) of longitude. When converting minutes to seconds, the conversion factor must be taken into account. 1'=60''!

The same applies to the calculation of the latitude:

$$y = \frac{1'}{7,4cm} \cdot 1,5cm = 0,2027' = 12''$$

The remaining distances calculated in this way are subtracted from the reference coordinates or added to them (depending on their position).

$$11^{\circ}30' - x = 11^{\circ}30' - 47'' = 11^{\circ}29'13'' \quad 50^{\circ}41' + y = 50^{\circ}41' + 12'' = 50^{\circ}41'12''$$

The geographical coordinates you are looking for are: 11°29'13'' eL und 50°41'12'' nW.

GK coordinates:

For the determination of the GK coordinates, the measured residual distances are converted into natural dimensions.

$$x = 0,9cm \cdot 25000 = 22500cm = 225m \quad y = 3,2cm \cdot 25000 = 80000cm = 800m$$

The GK coordinates we are looking for are: R 4 401 225m und H 5 538 800m.

UTM coordinates

The Universal Transversal Mercator (UTM) system, like the Gauß-Krüger system, belongs to the geodetic map projections, i.e. the mapped area is relatively small in order to keep distortions low. By systematically shifting the mapping area, the whole earth can be imaged in UTM coordinates, except for the polar regions.

The UTM image uses a transversal section cylinder as the imaging body (see Figure 10). In contrast to the Gauß-Krüger image, which works with a touch cylinder, the UTM image allows a larger area to be imaged in a coordinate system (a meridian strip) without exceeding the permissible distortions.

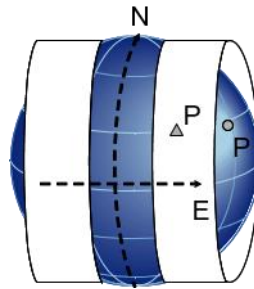


Figure 10: Mapping principle of the UTM mapping

Both the Gauß-Krüger image and the UTM image are conformal, i.e. angles (and thus also shapes) are not distorted. This property is very important for navigation maps, since course lines (constant angle => Loxodrome) can be entered as straight lines in maps with conformal mapping. The underlying projection is named after its developer Gerhardus Mercator, the Mercator projection (UTM = Universal Transversal Mercator). Mercator used a pole standing cylinder for his maps, which leads to the fact that the poles cannot be illustrated and pole near areas are very strongly distorted. This becomes particularly clear when comparing Greenland with Africa. In a map with Mercator projection Greenland and Africa appear almost the same size. In fact, Greenland has only about 1/14 part of the area of Africa. The Mercator projection is therefore not true to area (see Figure 11).

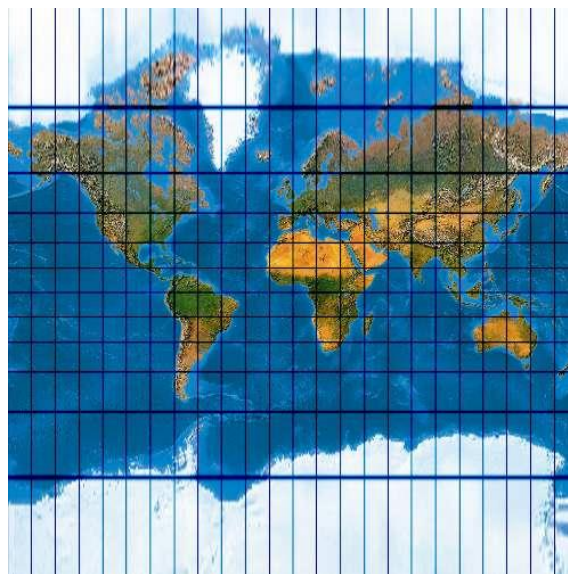


Figure 11: World map in MERCATOR projection

(Source: www.boehmwanderkarten.de/kartographie/is_netze_cyl.html)

The UTM image and the Gauss-Krüger image take advantage of the fact that the Mercator projection images areas along the contact meridian (or intersection meridians) with virtually no distortion. For Mercator, this concerned areas along the equator. In order to be able to use this advantage for areas further north and further south, the pole cylinder of Mercator is converted into a transverse cylinder. This cylinder touches (Gauß-Krüger) resp. cuts (UTM) the earth shape along one resp. two meridians. The favourable distortion properties thus exist for areas along this meridian. The geographical latitude is no longer important. The contact meridian or the intersection meridians is or are mapped true to length when the cylinder is unwound into the plane.

With increasing distance from the contact meridian or from the intersection meridians, the distortion increases as well as with the Mercator projection with increasing distance from the equator. Therefore, the validity of the image is limited to a strip along the contact meridian or the intersection meridians (the meridian strip). For geodetic applications (purposes of land surveying and cadastral surveying), corresponding corrections must also be made within the meridian strip (distance and area corrections).

In order to be able to represent all areas of the earth by means of the UTM image, the cylinder "wanders" step by step around the earth. The middle meridians of the displayed meridian stripes are fixed. In the UTM image, every sixth meridian (starting from 177° western longitude) is used as the middle meridian. The cut meridians are each 1.5° from the middle meridian.

The UTM coordinate system is a plane rectangular coordinate system whose ordinate (Easting = east value, distance from the middle meridian) runs along the equator and whose abscissa (Northing = north value, distance from the equator) runs perpendicular to it along the middle meridian. The imaging range of the UTM coordinate system is limited to 84° northern latitude and 80° southern latitude. The different extent in north-south direction is due to the different location of "important" land areas. Greenland, Canada and Russia extend beyond 80° northern latitude, while only Antarctica lies south of 80° southern latitude. A further problem at high latitudes makes it necessary to limit the imaging range – the meridian convergence. In the vicinity of the poles, for example, the imaging area (the extent of the meridian stripe) goes towards zero. Since the Northing value (like the high value in the Gauß-Krüger image) refers to the measurement along the mid meridian with a scale factor of 0.9996, the curvature of the meridians is not taken into account in the coordinates. The coordinate axis of the Northing values is perpendicular to the coordinate axis of the Easting values that runs along the equator (see Figure 12). This makes the deviations near the poles very large. The curvature of the meridians, i.e. the angle between the meridian and the grid north, is greatest at the poles. The polar regions are represented by more favourable images, e.g. the UPS image (Universal Polar Stereographic).

The meridian stripes of the UTM image are 6° wide. Neighbouring stripes overlap by 0.5°, so that the individual stripes are actually 7° wide. The middle meridians of the meridian stripes are fixed. In the Central European region the meridians are at 3°, 9° and 15° (Austrian).

The range between the intersection meridians is compressed by the scale factor of the middle meridian (0.9996). In the UTM image, for example, the greatest distortions occur at the mid meridian. There it is about 40 cm at a distance of 1 km. The scale factor for the compressed areas is between 0.9996 and 1. The areas between the respective boundary meridian and the intersection meridian are stretched in the UTM image. The scale factor for these areas is greater than 1. The intersection meridians are mapped true to length (see Figure 13).

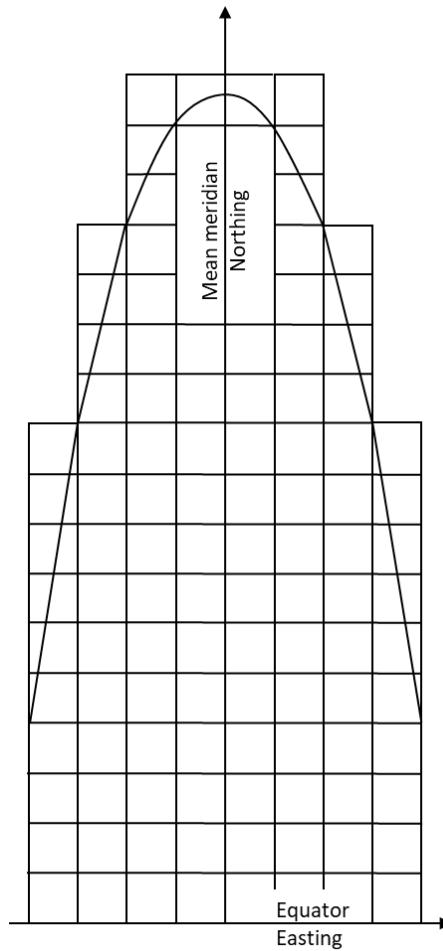


Figure 12: Meridian convergence and coordinate grid

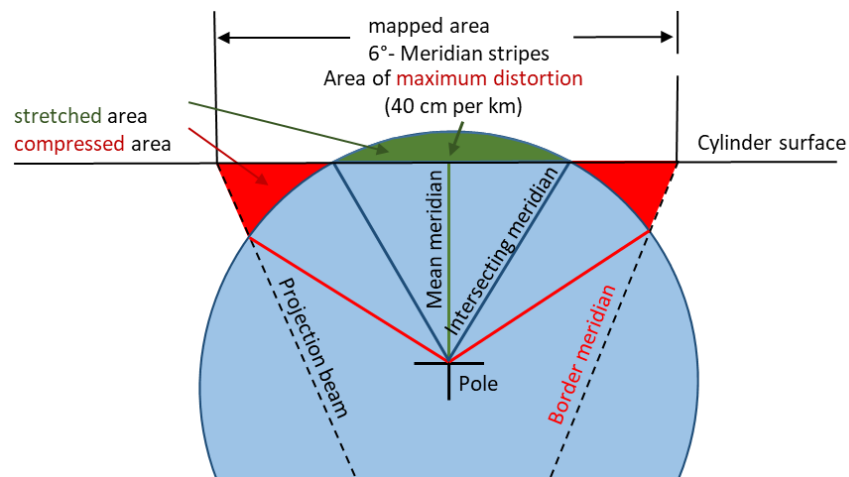


Figure 13: Distortion ranges of the UTM projection

Similar to the Gauss-Krüger coordinates, the UTM coordinates are called Easting (corresponds to the right value) and Northing (corresponds to the high value). However, Gauss-Krüger coordinates and UTM coordinates can never be equated. The UTM coordinates correspond to a Gauß-Krüger image

with 6° meridian stripes using the scale factor 0.9996. As in the Gauß-Krüger coordinate system, the middle meridian in the UTM coordinate system also receives the easting value 500 000 m.

In topographic maps the conversion from Gauss-Krüger to UTM coordinates took place. Both coordinate systems can be clearly distinguished on the basis of the given right and easting values. While the right-hand values of the Gauß-Krüger coordinates are given at the edge of the map with four digits (strip number + distance, km accuracy), the easting value has only three digits (km accuracy). The corresponding meridian strip is indicated in the UTM coordinate system by the zone number. It is usually not possible to distinguish between the easting and northing values, since the two values are very similar.

A typical indication of UTM coordinates in topographic maps is:

E ³57 000m N ⁵⁴89 000m

The zone number is indicated at the edge of the map. The zone number of the meridian strip with the middle meridian 15° is 33.

In some cases, the easting values are also given a strip number. This is identical to the strip numbers of the Gauß-Krüger system with 6° meridian stripes, i.e. the strip with the middle meridian 15° is assigned the strip number 3. The easting value of the above coordinate could therefore also be given as follows: E 3 357 000 (in m). The zone number is therefore not given.

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http://www.boehmwanderkarten.de/kartographie/is_netze_cyl.html: comprehensive overview of map net designs and projections

<http://www.kowoma.de/gps/geo/Projektionen.htm>: Information zu Koordinatensystemen, Projektionen und Kartennetzentwürfen.